A New Power Index and Its Axioms System for a Voting Game with Multialternatives (Mathematical Decision Making under Uncertainty)

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1. INTRODUCTION

In various collective decision makings many voting systems are adopted as a method of putting opinions of all members together. In such a case it is very important to get the influence of each member upon a collective decision making quantitatively because of the following reasons :

(1) We want to know the influence itself of each member in order to utilize it on the negotiation with other members.

(2) By looking over the power indices of all members, we want to make a fair judgment on the right or wrong of the voting system, that is, the voting system, in which the power indices are unfair, should be unused.

(3) When we intend to share the common profit or cost in proportion to the influence of each member, it is very important for each member to get the power indices correctly.

Various power indices have been proposed, for example,

(i) The Shapley-Shubik index	(the S-S index)	[5]
(ii) The Banzhaf index	(the B index)	[1]
(iii) The Coleman index	(the C index)	[3]
(iv) The Deegan-Packle index	(the D-P index)	[4]
(v) The Yamamoto-Nakai index	(the Y-N index)	[6]

Since these indices are presented with the axioms systems on which they are based, their backgrounds and properties are clear. The advantages and disadvantages of these indices can't be discussed theoretically, for they are independent mutually and have equivalent values. Then, applying one of them to the actual problem, it is desirable to decide which index should be used according as the actual conditions.

The above five indices are related to a voting system with respect to a choice between the ayes and nays, that is, they treat only the case of two alternatives. This fact is closely related to the definition of a characteristic function in a cooperative game by von Neumann and Morgenstern. Let N be the set of players. A characteristic function $v(S)$ is defined on the class of subsets of N and means the max−min value of the expected payoff of the coalition $S(\subseteq N)$ in a two−person game by two coalitions S and $N-S$. In this situation there are only two alternatives for each player, that is, belonging to the coalition S and belonging to $N-S$.

On the other hand there are many voting systems with more than two alternatives, for example, the case that each member votes one of four policies proposed for resolving some problem. Furthermore it happens frequently that in the election of assemblymen we vote one of many candidates. In the case of considering the power of each player in a voting game with multialternatives we can't evaluate it correctly by the traditional characteristic function defined by von Neumann and Morgenstern. For example, we consider a simple majority game with seven voters $(a_1,\cdot\cdot\cdot,a_7)$ and three alternatives. Under the traditional characteristic function all coalitions composed of three voters are losing coalitions and therefore it is not evaluated at all affirmatively to participate in a coalition with three voters. But in practice if the division with three coalitions $\{a_1,a_2,a_3\}$, $\{a_4,a_5\}$, $\{a_6,a_7\}$ occurs, the coalition $\{a_1,a_2,a_3\}$ wins though it is composed of three voters. Then the traditional characteristic function can't evaluate correctly the power of each voters. In the case of multialternatives, the relationship of cooperation among players becomes complicated and a coalition is formed for each alternative. As a result, the set N is divided into subsets $N_1,\cdots,N_r$ where N_i is the set of supporters for alternative i and a characteristic function is denoted by a vector function $\{v_1(N_1),\cdots,v_r(N_r)\}$ where $v_i(N_i)$ means the expected payoff of the coalition N_i under the division $\{N_1,\cdots,N_r\}$. In this way, the extension of the concept of the traditional characteristic function is required.

With respect to a simple game with multialternatives, Bolger [2] proposes a power index, together with its axioms system. The Bolger index is based on a similar idea to the Shapley value and does not treat a weighted voting game. In this paper we propose another power index which is an extension of the Deegan-Packle index to the case of multialternatives and contains the case of a weighted voting game. In Section 2 the concept of a simple game with multialternatives is introduced and a new power index is proposed. In Section 3 we show an axioms system on which the new index is based.

2. A NEW POWER INDEX

Before proposing a new power index, we need to introduce some notions.

$N=\{1,2,\cdots,n\}$: a set of players (voters)

$R=\{1,2,\cdots,r\}$: a set of alternatives

We assume that each player votes only one alternative. As a result, the set N is divided to r subsets. Let $C=\{C(1),\cdots,C(r)\}$ be a division of the set N satisfying

$$C(j)\cap C(k)=\phi \quad (j\neq k) \quad , \quad \bigcup_{j=1}^{r} C(j)=N$$

where $C(j)$ is the set of supporters for alternative j and is called a coalition. The number of divisions is r^n.

An electoral system is a rule of deciding a successful alternative from the result of voting. When a division $C=\{C(1),\cdots,C(r)\}$ occurs and alternative j is elected, the coalition $C(j)$ is called a winning coalition (WC) and denoted by $W(C)$. In general many winning coalitions may exist, for example, there is a case that some alternatives in a higher rank are elected. But in this paper we concentrate our consideration on the case that the number of successful alternatives is one at most. A winning coalition $W(C)$ should satisfy the following two conditions :

(i) If $C(j)=N$, then $C(j)=W(C)$. That is to say, if all players cooperate, they can certainly win. Then under any electoral system there exists at least one division having a WC.

(ii) The monotonicity: If a WC only expands and other coalitions don't expand, then the expanded coalition is also a WC for the new division. Going into details, when $W(C)=C(j)$ under a division C, we consider a new division $C'=\{C'(1),\cdots,C'(r)\}$ where $C'(j)\supset C(j)$ and $C'(k)\subseteq C(k)$ for any $k(\neq j)$. So under the new division C', $C'(j)=W(C')$.

We consider a weighted voting game in which each player has the different number of votes mutually. Let $d(i)$ be the number of votes belonging to player i. If we put

$$\|C(j)\|=\sum_{i\in C(j)} d(i), \tag{1}$$

then this denotes the number of votes obtained by alternative j.

Case1 : A WC is defined as one selected randomly from coalitions maximizing the value of $\|C(j)\|$. In this case a WC always exists.

Case2 : A WC is defined as the coalition $C(j)$ satisfying

$$\|C(j)\|>\frac{1}{2}\sum_{k=1}^{r}\|C(k)\|, \tag{2}$$

that is, a WC has a majority of the total number of votes. In this case it may occur that there is no WC.

Definition 1. A characteristic function is defined as an r−dimensional vector function $v(C) = \{v_1(C), \cdots, v_r(C)\}$ on the set of divisions.

Definition 2. 3-tuple (N, R, v) is a simple game with multialternatives if and only if

$$v_j(C) = \begin{cases} 1 & \text{if } C(j) = W(C) \\ 0 & \text{if } C(j) \neq W(C) \end{cases} \qquad (j = 1, \cdots, r). \tag{3}$$

When a division $C = \{C(1), \cdots, C(r)\}$ occurs, the value of $v_j(C)$ means a payoff of the coalition $C(j)$. In a simple game a WC only can obtain a payoff one and other coalitions can obtain payoff zero. In a simple game (N, R, v), if two sets N and R are fixed, we omit them and indicate the game by the characteristic function v only. Selecting one electoral system means deciding a characteristic function and furthermore corresponds to deciding a WC.

Definition 3. When a coalition $C(j)$ is a WC for a division $C = \{C(1), \cdots, C(r)\}$, $C(j)$ is a minimal winning coalition (MWC) if and only if for any $h(\in C(j))$, there is an appropriate alternative l ($l \neq j$ and l depends on h) such that $\tilde{C}(j) \neq W(\tilde{C})$ for a division $\tilde{C} = \{\tilde{C}(1), \cdots, \tilde{C}(r)\}$ where

$$\tilde{C}(k) = \begin{cases} C(j) - \{h\} & k = j \\ C(l) \cup \{h\} & k = l \\ C(k) & k \neq j, l. \end{cases} \tag{4}$$

Definition 3 states that any player in the MWC $C(j)$ can make alternative j be defeated in the election by changing his support from alternative j to another one. That is, alternative j cannot be elected if he is betrayed by someone in the MWC $C(j)$. Then as an indispensable member of the MWC any member in the MWC can assert the same right as other members. As a result, the common profit one for the MWC comes to be divided equally among all members in the MWC.

When a WC $C(j)$ is given, excluding all non−indispensable members for the success of alternative j from $C(j)$, we can obtain a MWC which is called an induced MWC from a WC $C(j)$ and indicated by $I[C(j)]$.

G : a set of simple games with common sets N and R
$W_v(C)$: a WC for a division C under a simple game v
$M_v(C)$: a MWC for a division C under a simple game v
$\Gamma(v)$: a set of divisions with a MWC under a simple game v
$\Gamma_i(v)$: a set of divisions with a MWC including player i under a simple game v

Our new power index is derived from the following five fundamental assumptions regarding the behavior of players :

(i) Only MWC's will emerge victorious.
(ii) Each division with a MWC has an equal probability of forming.
(iii) For a fixed division, a MWC only can obtain the payoff one and the payoffs of other coalitions are zero.
(iv) The common profit of a MWC is divided equally among all members in the MWC.
(v) The power index for each player is proportional to his expected payoff.

We propose a new power index based on the above five assumptions as follows :

$$\rho_i(v) = \frac{1}{|\Gamma(v)|} \sum_{C \in \Gamma_i(v)} \frac{1}{|C(\beta_i)|} \qquad (i = 1, \cdots, n) \tag{5}$$

where $|A|$: the number of elements of a set A

β_i : an alternative which player i supports.

The index $\rho_i(v)$ denotes the degree of influence of player i upon the collective decision making under a simple game v with multialternatives. Note that the denominators of the equation (5) are not zero.

3. THE AXIOMS SYSTEM

Though the new index $\rho_i(v)$ is derived from the idea of the above five assumptions, in order to clarify its backgrounds and properties we shall show an axioms system on which the new index is based. Before showing it, we need some definitions and lemmas.

Definition 4. In a simple game (N, R, v) with multialternatives, we consider a division

$C=\{C(1),\cdots,C(r)\}$. For any permutation σ on N, two definitions are introduced.

(i) A new pseudo division : $\sigma^{-1}(C)=\{\sigma_1^{-1}(C),\cdots,\sigma_r^{-1}(C)\}$

where $\sigma_j^{-1}(C)=\{i \mid \sigma(i)\in C(j)\}$.

Note that $\sigma^{-1}(C)$ is not an ordinary division because all members of $\sigma^{-1}(C)$ don't necessarily support the alternative j. We define a winning coalition for the pseudo division $\sigma^{-1}(C)$ as follows :

$$\sigma_j^{-1}(C)=W_v(\sigma^{-1}(C)) \Leftrightarrow C(j)=W_v(C).$$

(ii) A new simple game σv : $(\sigma v)(C)=\{(\sigma v)_1(C),\cdots,(\sigma v)_r(C)\}$

$$\text{Where} \quad (\sigma v)_j(C)=v_j(\sigma^{-1}(C))=\begin{cases}1 & \text{if } \sigma_j^{-1}(C)=W(\sigma^{-1}(C))\\ 0 & \text{otherwise.}\end{cases} \tag{6}$$

Definition 5. Two games v and $w(\in G)$ are mergeable if and only if $\Gamma(v)\cap\Gamma(w)=\phi$.

Therefore if v and w are mergeable, then it is impossible that there exist alternatives j and k satisfying $C(j)=M_v(C)$ and $C(k)=M_w(C)$ simultaneously, including the case of $j=k$.

Definition 6. When two games v and $w(\in G)$ are mergeable, a combining game $v\vee w$ is defined as follows : for a division $C=\{C(1),\cdots,C(r)\}$,

$$(v\vee w)(c)=\{(v\vee w)_1(C),\cdots,(v\vee w)_r(C)\}$$

where

$$(v\vee w)_j(C)=\begin{cases}1 & \text{if } C(j)=W_{v\vee w}(C)\\ 0 & \text{otherwise}\end{cases} \tag{7}$$

and

$$W_{v\vee w}(C)=\begin{cases}\text{doesn't exist} & \text{if both } W_v(C) \text{ and } W_w(C) \text{ don't exist}\\ C(j) & \text{if } W_v(C)=C(j) \text{ and } W_w(C) \text{ doesn't exist}\\ C(k) & \text{if } W_w(C)=C(k) \text{ and } W_v(C) \text{ doesn't exist.}\end{cases} \tag{8}$$

Remark 1. (i) Since v and w are mergeable, it is impossible that both $W_v(C)$ and $W_w(C)$ exist.
(ii) The relation, which is obtained by exchanging W (winning coalition) with M (minimal winning coalition) in the equation (8), is valid.

LEMMA 1. If two games v and $w(\in G)$ are mergeable, then

(i) $\Gamma(v \vee w) = \Gamma(v) + \Gamma(w)$ (9)

(ii) $\Gamma_i(v \vee w) = \Gamma_i(v) + \Gamma_i(w)$ $(i = 1, \cdots, n)$. (10)

PROOF. We prove the assertion (i) only since the assertion (ii) can be also proved similarly. From Remark 1 (ii), we obtain

$$M_{v^\vee w}(C) = M_v(C) \text{ or } M_w(C)$$

and therefore

$$C \in \Gamma(v \vee w) \Leftrightarrow C \in \Gamma(v) \bigcup \Gamma(w).$$

On the other hand, because of the mergeability $\Gamma(v) \bigcap \Gamma(w) = \phi$.
Then $\Gamma(v \vee w) = \Gamma(v) + \Gamma(w)$.

Under a simple game v, let L_v be the set of coalitions which is a MWC for some division, that is,

$$L_v = \{M_v(C) \mid C \in \Gamma(v)\}. \tag{11}$$

Numbering all elements of the set L_v, we denote them by $M_v^1, \cdots, M_v^m$ where $m = |L_v|$.

Definition 7. For a simple game v, singleton games $v^k (k = 1, \cdots, m)$ are defined as follows : For a division $C = \{C(1), \cdots, C(r)\}$,

$$v^k(C) = \{v_1^k(C), \cdots, v_r^k(C)\} \qquad (k = 1, \cdots, m)$$

where

$$v_j^k(C) = \begin{cases} 1 & \text{if } C(j) = W_v(C)\ ,\ I[W_v(C)] = M_v^k\ , \\ 0 & \text{otherwise.} \end{cases} \tag{12}$$

Under a simple game v we consider a division $C=\{C(1),\cdots,C(r)\}$. If $C(j)=W(C)$ and $I[C(j)]=M_v^l$, then

$$v^l(C)=\{0,\cdots,0,\overset{\overset{j}{\vee}}{1},0,\cdots,0\} \tag{13}$$

and

$$v^k(C)=\{0,\cdots,0\} \qquad \textit{for any } k(\neq l). \tag{14}$$

Furthermore we obtain

$$M_{v^k}(C)=\begin{cases} M_v^k & \textit{if } C\in\Gamma(v^k) \\ \textit{doesn't exist} & \textit{otherwise.}\end{cases} \tag{15}$$

LEMMA 2. Let $v^1,\cdots,v^m$ be singleton games for a simple game v.

(i) $v^1,\cdots,v^m$ are mergeable mutually.

(ii) A simple game v is a combining game of its singleton games, that is , $v=v^1\vee v^2\vee\cdots\vee v^m$.

PROOF. (i) Since $\Gamma(v^j)$ is the set of divisions with a *MWC* M_v^j, it is clear that $\Gamma(v^j)\cap\Gamma(v^k)=\phi$ $(j\neq k)$. Then $v^1,\cdots,v^m$ are mergeable.

(ii) $v(C)=\{v_1(C),\cdots,v_r(C)\}$

$$\text{where } v_j(C)=\begin{cases}1 & \textit{if } C(j)=W_v(C)\\ 0 & \textit{otherwise.}\end{cases} \tag{16}$$

On the other hand,

$$(v^1\vee\cdots\vee v^m)(C)=\{(v^1\vee\cdots\vee v^m)_1(C),\cdots,(v^1\vee\cdots\vee v^m)_r(C)\}$$

$$\text{where } (v^1\vee\cdots\vee v^m)_j(C)=\begin{cases}1 & \textit{if } C(j)=W_{v^1\vee\cdots\vee v^m}(C)\\ 0 & \textit{otherwise.}\end{cases} \tag{17}$$

Furthermore by the equation(8),

$$W_{v^1\vee\cdots\vee v^m}(C)=\begin{cases} W_{v^k}(C) & \textit{if only } W_{v^k}(C) \textit{ exists} \\ \textit{doesn't exist} & \textit{if no } W_{v^k}(C) \textit{ exists.} \end{cases} \tag{18}$$

(Because of the mergeability of $v^1,\cdots,v^m$, more than one $W_{v^k}(C)$ can't exist simultaneously.)

Then from (16), (17) and (18) the result (ii) can be obtained.

LEMMA 3. Let σ be any permutation on N and v^k be a singleton game for a simple game v. Then

$$\sigma v^k = v^k \qquad (k=1,\cdot\cdot\cdot,m). \tag{19}$$

PROOF.

$$(\sigma v^k)_j(C)=v_j^k(\sigma^{-1}(C))=\begin{cases} 1 & \textit{if} \quad \sigma_j^{-1}(C)=W_{v^k}(\sigma^{-1}(C)) \\ 0 & \textit{otherwise} \end{cases} \tag{20}$$

$$v_j^k(C)=\begin{cases} 1 & \textit{if} \quad C(j)=W_{v^k}(C) \\ 0 & \textit{otherwise} \end{cases} \tag{21}$$

The assertion of $\sigma_j^{-1}(C)=W_{v^k}(\sigma^{-1}(C))$ is equivalent to the assertion of $C(j)=W_{v^k}(C)$.

Then from (20) and (21), we obtain $\sigma v^k = v^k \quad (k=1,\cdot\cdot\cdot m)$.

We propose a new axioms system which a power index should satisfy.

Let $\pi(v)=\{\pi_1(v),\cdot\cdot\cdot,\pi_n(v)\}$ be a power index vector for any game $v(\in G)$.

Axiom A_1: $\pi_i(v)=0$ if and only if $\Gamma_i(v)=\phi$.

(The power index is zero if and only if he is a dummy player.)

Axiom A_2: For any permutation σ on N,

$$\pi_{\sigma(i)}(\sigma v)=\pi_i(v) \quad \text{for any } i(\in N) \text{ and any } v(\in G). \tag{22}$$

(Changing names of players does not affect their powers.)

Axiom A_3:
$$\sum_{i=1}^{n}\pi_i(v)=1. \tag{23}$$

(The power index vector has the normalization property.)

Axiom A_4: If two games v and $w(\in G)$ are mergeable, then

$$\pi(v \vee w) = \frac{|\Gamma(v)|}{|\Gamma(v \vee w)|}\pi(v) + \frac{|\Gamma(w)|}{|\Gamma(v \vee w)|}\pi(w). \tag{24}$$

(The power index of the combining game is the convex linear combination of the power indices of its component games.)

LEMMA 4. If Axiom $A_1, A_2 and A_3$ are satisfied, then

$$\pi_i(v^k) = \begin{cases} \frac{1}{|\Gamma(v^k)|}\sum_{C\in\Gamma_i(v^k)}\frac{1}{|C(\beta_i)|} = \frac{1}{|M_v^k|} & \text{if } i \in M_v^k \\ 0 & \text{otherwise} \end{cases} \tag{25}$$

for any $i(\in N)$ and any singleton game v^k.

PROOF. The result is proved through four steps.

(i)When $i \notin M_v^k$, it is clear that $\Gamma_i(v^k) = \phi$, and therefore $\pi_i(v^k) = 0$ because of Axiom A_1.

(ii)When $i \in M_v^k$, we shall prove that $\pi_i(v^k)$ is independent of i ,that is,

$$\pi_j(v^k) = \pi_l(v^k) \quad \text{for any} \quad j,\ l \in M_v^k \quad (j \neq l). \tag{26}$$

We consider a permutation σ on N defined by

$$\sigma(i) = \begin{cases} l & i = j \\ j & i = l \\ i & i \neq j, l. \end{cases} \tag{27}$$

Then by Axiom A_2 and Lemma 3, we obtain

$$\pi_j(v^k) = \pi_{\sigma(j)}(\sigma v^k) = \pi_l(v^k) \quad \text{for any} \quad j, l \in M_v^k \quad (j \neq l). \tag{28}$$

(iii)When $i \in M_v^k$, omitting the subscript , we can replace $\pi_i(v^k)$ with $\pi(v^k)$ because of (ii). By Axiom A_1 and A_3, we obtain

$$\begin{aligned} 1 &= \sum_{i=1}^{n}\pi_i(v^k) = \sum_{i\in A}\pi_i(v^k) \\ &= \sum_{i\in M_v^k}\pi(v^k) = \pi(v^k)|M_v^k|. \end{aligned} \tag{29}$$

where $A = \{i|\Gamma_i(v^k) \neq \phi\}$. Then $\pi(v^k) = |M_v^k|^{-1}$.

(iv)When $i \in M_v^k$,

$$|\Gamma(v^k)|^{-1} \sum_{C \in \Gamma_i(v^k)} |C(\beta_i)|^{-1} = |\Gamma(v^k)|^{-1} \sum_{C \in \Gamma_i(v^k)} |M_v^k|^{-1}$$

$$= |\Gamma(v^k)|^{-1} \times |M_v^k|^{-1} \times |\Gamma(v^k)| = |M_v^k|^{-1}. \tag{30}$$

Then from (iii) and (iv), we obtain (25).

THEOREM 1. (i) The vector $\rho(v) = \{\rho_1(v), \cdot \cdot \cdot, \rho_n(v)\}$ given by (5) satisfies Axiom $A_1 \sim A_4$.

(ii) The power index vector satisfying Axiom $A_1 \sim A_4$ coincides with the vector $\rho(v)$ given by (5).

PROOF. (i) <Axiom A_1> : From the equation (5) the result is clear.

<Axiom A_2>:

$$\begin{aligned}\rho_{\sigma(i)}(\sigma v) &= |\Gamma(\sigma v)|^{-1} \sum_{C \in \Gamma_{\sigma(i)}(\sigma v)} |C(\beta_{\sigma(i)})|^{-1} \\ &= |\Gamma(v)|^{-1} \sum_{C \in \Gamma_i(v)} |C(\beta_i)|^{-1} \\ &= \rho_i(v).\end{aligned} \tag{31}$$

<Axiom A_3>:

$$\begin{aligned}\sum_{i=1}^{n} \rho_i(v) &= |\Gamma(v)|^{-1} \sum_{i=1}^{n} \sum_{C \in \Gamma_i(v)} |C(\beta_i)|^{-1} \\ &= |\Gamma(v)|^{-1} \sum_{C \in \Gamma(v)} |C(\beta_i)|^{-1} \times |C(\beta_i)| \\ &= 1\end{aligned} \tag{32}$$

<Axiom A_4>:

$$\begin{aligned}\rho_i(v \vee w) &= |\Gamma(v \vee w)|^{-1} \sum_{C \in \Gamma_i(v \vee w)} |C(\beta_i)|^{-1} \\ &= |\Gamma(v \vee w)|^{-1} \{ \sum_{C \in \Gamma_i(v)} |C(\beta_i)|^{-1} + \sum_{C \in \Gamma_i(w)} |C(\beta_i)|^{-1} \} \\ &= \frac{1}{|\Gamma(v \vee w)|} \left\{ \frac{|\Gamma(v)|}{|\Gamma(v)|} \sum_{C \in \Gamma_i(v)} |C(\beta_i)|^{-1} + \frac{|\Gamma(w)|}{|\Gamma(w)|} \sum_{C \in \Gamma_i(w)} |C(\beta_i)|^{-1} \right\} \\ &= \frac{|\Gamma(v)|}{|\Gamma(v \vee w)|} \rho_i(v) + \frac{|\Gamma(w)|}{|\Gamma(v \vee w)|} \rho_i(w)\end{aligned} \tag{33}$$

where the second equality is derived from Lemma 1.

(ii) $\pi_i(v)$

$$= \pi_i(v^1 \vee \cdot \cdot \cdot \vee v^m) \qquad \text{(by Lemma 2)}$$

$$= |\Gamma(v)|^{-1} \sum_{k=1}^{m} |\Gamma(v^k)| \pi_i(v^k) \qquad \text{(by Axiom } A_4\text{)}$$

$$= |\Gamma(v)|^{-1} \sum_{k=1}^{m} \sum_{C \in \Gamma_i(v^k)} |C(\beta_i)|^{-1} \qquad \text{(by Lemma 4)}$$

$$= |\Gamma(v)|^{-1} \sum_{C \in \sum_{k=1}^{m} \Gamma_i(v^k)} |C(\beta_i)|^{-1}$$

$$= |\Gamma(v)|^{-1} \sum_{C \in \Gamma_i(v^1 \vee \cdot \cdot \cdot \vee v^m)} |C(\beta_i)|^{-1} \qquad \text{(by Lemma 1)}$$

$$= |\Gamma(v)|^{-1} \sum_{C \in \Gamma_i(v)} |C(\beta_i)|^{-1} \qquad \text{(by Lemma 2)} \qquad (34)$$

$$= \rho_i(v).$$

By Theorem 1, we know that the new index is a unique vector function satisfying the above axioms system.

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